DCI-003-1164001

Seat No. ______

M. Sc. (Sem. IV) Examination

July - 2022

Mathematics : CMT-4001

(Linear Algebra)

Faculty Code : 003
Subject Code : 1164001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :
(1) There are total five questions.
(2) All questions are mandatory.
(3) Each question carries equal marks.

1 Answer any seven of the following questions :

(1) Define with example :

(a) Left-invertible linear transformation.

(b) Right-invertible linear transformation.

(2) Define with example : Characteristic root.

(3) Define with example : Nilpotent linear transformation.

(4) Define with example : Cyclic subspace with respect to T.

(5) Define with example : Basic Jordan block belonging to λ.

(6) Define Rational Canonical Form.

(7) State Primary Decomposition Theorem.

(8) Define with example : Trace of a matrix.

(9) Define with example : Bilinear form.

(10) Define with example : Secular Equation.

2 Answer any two of the following questions :

(1) Let V be an n-dimensional vector space over $\mathbb{F}$ and $T \in A_{\mathbb{F}}(V)$. Prove that, T is singular if and only if there exists $v \neq 0$ in V such that $T(v) = 0$.

(2) Let $\mathbb{F}$ be a subfield of K. Let $n \in \mathbb{N}$ and $A \in \mathbb{F}_n$. Prove that A is invertible in $\mathbb{F}_n$ if and only if A is invertible in K_n.

(3) Let V be a finite dimensional vector space over $\mathbb{F}$ and $T, S \in A_{\mathbb{F}}(V)$. Prove that, if S is regular then T and STS^{-1} have the same minimal polynomial.

3 Answer the following questions :

(1) Let V be a finite dimensional vector space over $\mathbb{F}$ and $T \in A_{\mathbb{F}}(V)$. Prove that, T is regular if and only if T maps V onto V.

(2) Let V be a finite dimensional vector space over $\mathbb{F}$ and $T \in A_{\mathbb{F}}(V)$. Let T has all its characteristics roots in $\mathbb{F}$. Prove that, there exists a basis of V in which the matrix of T is triangular.

OR

3 Answer the following questions :

(1) Let V be an n-dimensional vector space over $\mathbb{F}$ and $T \in A_{\mathbb{F}}(V)$. Let $m_1(T)$ be the matrix of T in the basis $\{v_1, v_2, \ldots\ldots\ldots, v_n\}$ and $m_2(T)$ be the matrix of T in the basis $\{w_1, w_2, \ldots\ldots\ldots, w_n\}$ over $\mathbb{F}$. Prove that, there exists $C \in \mathbb{F}_n$ such that $m_2(T) = Cm_1(T)C^{-1}$.

(2) Let the matrix $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \in \mathbb{R}_3$. Prove that, A is nilpotent and find the invariants of A.

4 Answer the following questions :

(1) Le $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix} \in \mathbb{R}_3$. Determine the Jordan form of A.

(2) Let $A, B \in \mathbb{F}_n$. If A' denotes the transpose of A, then prove that,

(i) $(A')' = A$

(ii) $(A+B)' = A' + B'$

(iii) $(AB)' = B'A'$

5 Answer any two of the following questions :

(1) Let $A \in \mathbb{F}_n$. If any two rows of A are identical, then prove that, $\det(A) = 0$.

(2) Using Crammer's rule find the solutions, in the real field, of the system of equations given below :

$$x + 2y + z = 3$$
$$2x + 3y + z = 4$$
$$x - y - z = 0$$

(3) Let V be a finite dimensional inner product space over $\mathbb{C}$. Let $T \in A_{\mathbb{F}}(V)$. If $\langle T(v), v \rangle = 0, \forall v \in V$, then prove that, $T = 0$.

(4) Let V be an n-dimensional inner product space over $\mathbb{C}$. Prove that, T is unitary if and only if $\langle T(u), T(u) \rangle = (u, u), \ \forall u \in V$.